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Interferometry-free protocol for demonstrating topological order

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We propose a protocol to demonstrate the topological order of a spin-1/2 lattice model with four-body interactions. Unlike other proposals, it does not rely on the controlled movement of quasiparticles, thus eliminating the addressing, decoherence, and dynamical phase problems related to them. Rather, the protocol profits from the degeneracy of the ground state. It involves the addition of Zeeman terms to the original Hamiltonian that are used to create holes and move them around in the system.

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I. INTRODUCTION

The notion of topological order (TO) has gradually become a new and relevant topic in condensed-matter physics.[1,2] It gives rise to a new paradigm of quantum phases of matter which are endowed with long-range correlations that cannot be detected by local order parameters.[3,4] This is a new feature not associated with the spontaneous breaking of a symmetry. Instead, the detection of these new phases involves nonlocal order parameters that reflect the global nature of these new highly strongly correlated systems. Similarly, TOs turn out to be of great interest in quantum information since they are considered as a resource of robustness against the decoherence that typically affects all quantum systems when we try to manipulate them with ease and control.[5] The possibilities range from quantum memories for storage of quantum states[6] to quantum computers capable of performing a set of universal quantum operations.[7–9] The underlying mechanism for this robustness arises in a typical scenario where the possible errors in the system are local, while quantum logical operations are nonlocal and thus potentially resilient to decoherence.

A practical way of describing a TO is as a strongly correlated system with a quantum lattice Hamiltonian with the following properties: (i) there is an energy gap between the ground state and the excitations; (ii) the ground state is degenerate; (iii) this degeneracy cannot be lifted by local perturbations. These features reflect the topological nature of the system. In addition, a signature of the TO is the dependence of that degeneracy on topological invariants of the lattice where the system is defined, such as Betti numbers.[10] When the system is placed onto an infinite plane, which has trivial topology, then the TO manifests itself through the nontrivial braiding properties of their quasiparticle excitations:[11] when two identical particles are exchanged on the plane, their common wave function picks up a nontrivial statistical phase. More generally, when one particle completely encircles another particle, the state of the system picks up a phase factor that is only trivial for bosons and fermions, otherwise they are Abelian[12,13] or non-Abelian anyons.[14–16] Thus, braiding statistics is also a signature of TO that can be tried experimentally. Other signatures such as the topological entanglement entropy have also been proposed recently.[17,18]

There has been a number of interesting experiments in order to detect braiding statistics[19–21] in fractional quantum Hall-effect systems. This has turned out to be more elusive than detecting fractional charge.[22] Thus, a number of experimental proposals has been introduced aiming at providing additional signatures of braiding statistics[23–27] in fractional quantum Hall systems, both Abelian and non-Abelian, which in turn would imply TO. For non-Abelian gauge theories, it is also possible to detect anomalous braiding statistics by interferometric means.[28,29] There exist such intereferometric proposals for the surface code introduced by Kitaev.[30,31] This is the system in which we are interested here.

In this paper we propose an alternative route to detect TO directly and without having to resort to interferometry of quasiparticles to probe their nontrivial braiding statistics. We use the fact that the ground-state degeneracy is sensitive to the topology of the surface, which we can alter introducing Zeeman terms in certain areas of the system. In particular, our scheme for detecting TO relies on the notion of code deformations for surface codes.[6,32,33]

II. MODEL WITH STRING CONDENSATION

A. Hamiltonian and ground state

The topologically ordered system that we consider here was introduced by Kitaev.[5] It is a two-dimensional array of spin-1/2 systems. Note that any subset C of the spins can be identified with a binary vector (e_i), where $e_i=1$ if the ith spin belongs to C and $e_i=0$ otherwise. Then, for each such set C we introduce the operators

$$X^C := \bigotimes_i \sigma_X^{e_i}, \quad Z^C := \bigotimes_i \sigma_Z^{e_i}. \tag{1}$$

Spins are located at the sites of a "chessboard" lattice (see Fig. 1). The Hamiltonian is a sum of plaquette operators X^p, Z^p which depend on the coloring of the plaquette p, dark or light,

$$H = -\sum_{p\in\mathcal{P}_D} g_p X^p - \sum_{p\in\mathcal{P}_L} g_p Z^p, \tag{2}$$

where $g_p>0$ is the coupling constant at plaquette p, $\mathcal{P}_D$ ($\mathcal{P}_L$) is the set of dark (light) plaquettes and we identify each plaquette with the set of spins in its corners. The spectrum of plaquette operators is $\{1,-1\}$ and they commute, so that the ground state is defined by the conditions

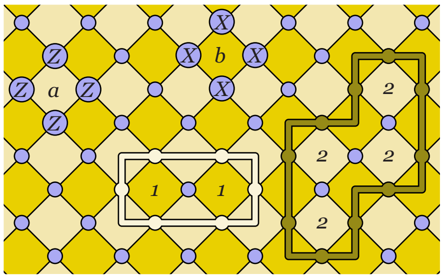

FIG. 1. (Color online) Blue circles represent the spin-1/2 systems, lying on the sites of the lattice. Z^p (X^p) operators correspond to light (dark) plaquettes such as a (b). The light (dark) string represents the product of the plaquette operators of those dark (light) plaquettes marked with a 1 (2).

$$X^p|\psi\rangle = Z^{p'}|\psi\rangle = |\psi\rangle, \quad p \in \mathcal{P}_D, p' \in \mathcal{P}_L, \tag{3}$$

which must hold for all the plaquettes. If we consider that the lattice extends to infinity or lies on a sphere, there is no ground-state degeneracy. In particular, the un-normalized ground state takes the form

$$|\mathrm{GS}\rangle = \prod_{p\in\mathcal{P}_D} (1 + X^p)|\psi_0\rangle, \tag{4}$$

where ψ_0 is the state with all spins up. However, if the topology of the surface is nontrivial the ground state is degenerate.[5]

B. String operators

A useful notion is that of dark and light strings, see Fig. 1 for examples. Light (dark) strings connect light (dark) plaquettes, so that each string segment contains a spin. Let γ be a light string and γ' a dark one. Then we attach string operators to them, X^γ and $Z^{\gamma'}$, identifying strings with the sets of spins in their segments. An important property is that $\{X^\gamma, Z^{\gamma'}\}=0$ if γ crosses γ' and odd number of times, $[X^\gamma, Z^{\gamma'}]=0$ otherwise. Strings are either closed or have end points at plaquettes of their color. When γ and γ' are closed we have

$$[X^\gamma, H] = [Z^{\gamma'}, H] = 0. \tag{5}$$

Among closed strings we find boundary strings, which receive this name because they form the boundary of a portion of the surface. Ground states can be characterized by the fact that if γ and γ' are boundaries then

$$X^\gamma|\psi\rangle = Z^{\gamma'}|\psi\rangle = |\psi\rangle. \tag{6}$$

This is equivalent to Eq. (3) because plaquettes can be identified with small boundaries, and boundary string operators are products of plaquette operators. We can also rewrite Eq. (4) as

$$|\mathrm{GS}\rangle = \sum_{\gamma\in\mathcal{B}^L} X^\gamma|\psi_0\rangle, \tag{7}$$

where the elements of $\mathcal{B}_L$ are collections of boundary strings. If we identify each state $X^\gamma|\psi_0\rangle$ with a string configuration, that corresponds to γ, then the ground state is a coherent superposition of string states. This is why we say that the model is a string condensate.[11]

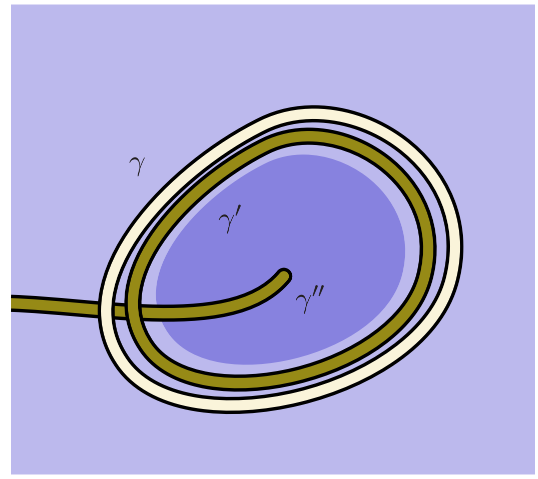

FIG. 2. (Color online) The total topological charge in the shaded region can be measured using the string operators X^γ and $Z^{\gamma'}$. String operators from strings with end points in the region, such as γ', change the charge of the region as they create or destroy a quasiparticle inside it.

C. Excitations and topological charge

The excitations of the system have a localized nature and are subject to an energy gap. In particular, these quasiparticles are related to plaquette operators, so that we say that the state $|\psi\rangle$ has an excitation at plaquette p if the corresponding condition (3) is violated. The energy of the quasiparticle is $\Delta=2g_p$. Excited states can be obtained from the ground state by applying open string operators: they create quasiparticles at their end points.

Excitations have a topological charge, which can be understood in terms of string operators also. Suppose that we have several excitations in the shaded region of Fig. 2. Consider a light string γ and a dark string γ' that surround the region. We construct four orthogonal projectors that resolve the identity

$$P_{a,b} \coloneqq \frac{1}{4}[1 + (-1)^a X^\gamma][1 + (-1)^b Z^{\gamma'}], \quad a,b = 0,1. \tag{8}$$

Each of the sectors (a,b) projected by $P_{a,b}$ corresponds to a different topological charge inside the region. These charges are integrals of motion because of Eq. (5). In the ground state the charge is (0,0), so this is the trivial charge. Consider a dark string γ'' with an end point inside the region, as in Fig. 2. Then we have $Z^{\gamma''}P_{a,b}=P_{a+1,b}Z^{\gamma''}$, with addition modulo two. Since γ'' switches the excitations of the dark plaquettes in its end points, we see that excitations of dark plaquettes carry the charge (1,0). Similarly, an excitation of a light plaquette carries the charge (0,1). It is easy to check that if a region is divided on two subregions with charges (a_1,b_1) and (a_2,b_2), then its total charge is (a_1+a_2,b_1+b_2), again with addition modulo two. The topological nature of these charges

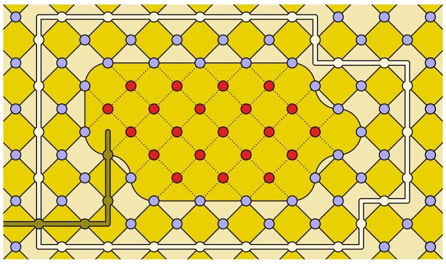

FIG. 3. (Color online) This figure represents a dark hole in a surface code. Red sites correspond to spins that are not part of the lattice. Note that a dark hole is nothing but a missing big dark plaquette. The strings on display are closed but not boundaries.

relies in the fact that when a charge (a_1,b_1) is moved around a charge (a_2,b_2) the system will pick up a phase $(-1)^{a_1b_2+a_2b_1}$ which does not depend on the particular trajectory.[5]

III. BORDERS AND TOPOLOGICAL DEGENERACY

A. Borders in surface codes

The ground-state subspace of the Hamiltonian (2) is a surface code, a kind of topological stabilizer code.[5] For our purposes here, a stabilizer code is a subspace defined by certain conditions, which for surface codes is Eq. (6). At first, surface codes were defined in closed surfaces, but this has a limited use since it is difficult to construct experimental setups with nonplanar geometries. However, even in the plane a nontrivial topology is possible if we introduce borders.[34,35]

Two kinds of borders can be considered in surface codes, dark or light. Borders change the concept of closed string. A dark (light) string is closed either if it has no end points or if its end points lie on dark (light) borders. Boundaries also change. A dark (light) string is a boundary if it encloses a portion of surface which contains no light (dark) borders. In surface codes, borders can be introduced by changing the geometry of the lattice. In particular, a dark (light) border corresponds to a missing big dark (light) plaquette. Then the code can be described using the conditions (6) under the new notion of boundary string. As an example, Fig. 3 shows a dark hole in a lattice. It has been created by erasing several spins from the lattice, shown in red, and rearranging the plaquettes accordingly.

The introduction of borders in surface codes allows to have nontrivial topologies and thus a code subspace with dimension greater than one. For example, if the surface is a disk with h holes, with the borders of the same type, then the dimension of the code is 2^h.[35] However, there is more to borders than this. In particular, we can consider adding dynamics to the picture. By changing the borders with time we can initialize, transform, and measure the states of the code.[33] This is a feature of surface codes that we would like to introduce in the quantum Hamiltonian model, a possibility that we explore next.

B. Borders in the string condensate

In principle, one could introduce borders in the quantum Hamiltonian model (2) simply by changing the geometry of the lattice, that is, as in the surface code of Fig. 3. However, this would require the ability to engineer a Hamiltonian in which for example a three-body plaquette term must exist next to a four-body one and so on. Such a detailed engineering is not feasible in many situations. Thus, we propose a different setting in which changes in the topology are produced by modifying the original Hamiltonian through the introduction of Zeeman terms and smooth spatial changes of the couplings.

We start by dividing the system surface in five regions, M, D, L, D_B, and L_B. M is the main system, where we are going to keep the original Hamiltonian and thus the topological order remains untouched. In the areas D and L there will be no topological order. As for D_B and L_B, these are thick boundaries that separate D from M and L from M, respectively. D_B will play the role of a dark boundary and L_B that of a light boundary. An example can be seen in Fig. 4, where the geometry is that of a disk with a hole, with both borders of light type.

We have to define the concepts of closed and boundary strings in our new geometry with the five regions. A dark (light) string is closed either if it has no end points or if they lie inside D (L). A dark (light) string is a boundary if it encloses a portion of surface not containing any piece of $L\cup L_B$ ($D\cup D_B$). With these definitions, we need a Hamiltonian that satisfies Eq. (5) for closed strings and such that its ground states satisfy Eq. (6) for boundary strings and there exists an energy gap to states not satisfying them. We will first show why these conditions are enough to get the desired properties and afterwards give an example of a Hamiltonian that satisfies the constrains.

We will work with a particular geometry to fix ideas, the disk with a hole of Fig. 4. Considering a general case has no additional complications, but the discussion would be less transparent. Let V be the subspace defined by conditions (6), so that the ground-state subspace is $V_{\mathrm{GS}}\subset V$. Consider the light string γ_1 and the dark string γ_1' of Fig. 4. They are closed but not boundaries, and since they cross we have $\{X^{\gamma_1},Z^{\gamma_1'}\}=0$. These operators are the X and Z operators of a qubit or two-level subsystem, both in V and in V_{GS}. Let us show this in detail. Note that X^{γ_1}, $Z^{\gamma_1'}$ leave V invariant, as closed string operators always commute with boundary string operators. Then we can choose an orthonormal basis $\{|0;k\rangle\}_k$ for the subspace of V such that $Z^{\gamma'}=1$, which can be completed in V with the elements $|1;k\rangle \coloneqq X^{\gamma_1}|0;k\rangle$, which satisfy $Z^{\gamma_1'}=-1$. In other words, $V\simeq V'\otimes V_2$, with V_2 a two-dimensional space. The same is true for V_{GS}, as follows from Eq. (5). That is, $V_{\mathrm{GS}}\simeq V_{\mathrm{GS}}'\otimes V_2$ and $V'\simeq V''\oplus V_{\mathrm{GS}}'$.

The point is that the degeneracy of the ground state that comes from the qubit subsystem has a topological origin and cannot be lifted by a small local perturbation. This is a consequence of the fact that there is an energy gap to states out of V and that if σ is any local operator then

$$\langle a;k|\sigma|b;k'\rangle = \delta_{a,b}\langle 0;k|\sigma|0;k'\rangle, \quad a,b=0,1. \tag{9}$$

We shall prove this equation in the following. For a local operator we mean one with a support such as the shaded area in Fig. 4, which neither encloses the central L region nor

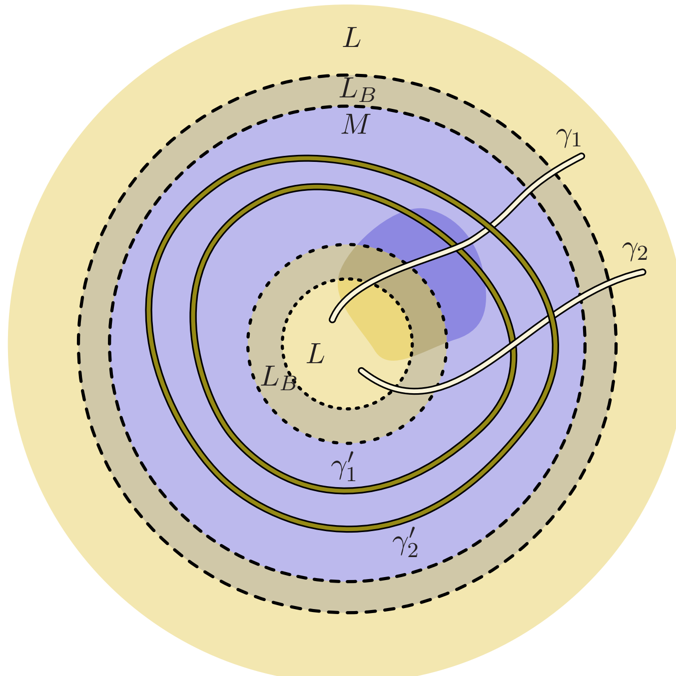

FIG. 4. (Color online) An example of how light borders are introduced in terms of the regions L, L_B, and M. In this case the main region M, in blue, has the topology of a disk with a hole, with both borders of light type. Examples of closed nontrivial strings are displayed. The γ_i are closed because they have no end points. The γ_i' are closed because their end points lie in L. The shaded area represents the support of a local operator, which neither encloses the interior L region nor connects both L regions.

connects the interior and exterior L regions. Then there exists a light string γ_2 and a dark string γ_2', as in the figure, with the following properties: First, they do not touch the support of σ, so that $[X^{\gamma_2},\sigma]=[Z^{\gamma_2'},\sigma]=0$. Second, we have the equivalences up to homology $\gamma_1\sim\gamma_2$, $\gamma_1'\sim\gamma_2'$, so that $X^{\gamma_1}X^{\gamma_2}=X^{\gamma_3}$ and $Z^{\gamma_1'}Z^{\gamma_2'}=Z^{\gamma_3'}$ with γ_3, γ_3' boundaries. From these properties Eq. (9) follows immediately. This equation can also be interpreted in terms of quantum error correction theory. It states that we can correct information codified in the qubit subsystem that has suffered a family of errors $\{E_i\}$ as long as any $\sigma=E_i^\dagger E_j$ is local.[36]

We now give an exactly solvable Hamiltonian that satisfies the desired constrains. It takes the form

$$H=-\sum_{p\in\mathcal{P}_D} g_pX^p-\sum_{p\in\mathcal{P}_L} g_pZ^p-\sum_i(\mu_iX^i+\nu_iZ^i), \qquad (10)$$

where i runs over the sites of the lattice, $g_p,\mu_i,\nu_i\geq 0$ are coupling constants, and we identify a site i with the set $\{i\}$. As long as $\nu_i=0$ ($\mu_i=0$) for all the sites i that lie on the corner of a dark (light) plaquette p with $g_p>0$, the Hamiltonian is exactly solvable because all the nonvanishing terms are commuting projectors. Then, the ground-state subspace is characterized by the conditions

$$X^p|\psi\rangle=Z^{p'}|\psi\rangle=Z^i|\psi\rangle=X^j|\psi\rangle=|\psi\rangle, \qquad (11)$$

which must hold for all the dark plaquettes p with $g_p>0$, light plaquettes p' with $g_{p'}>0$, sites i with $\nu_i>0$, and sites j with $\mu_j>0$. It is possible to choose the couplings in such a way that the conditions (5) and (6) are satisfied. In particular, $\mu_i>0$ ($\nu_i>0$) must be fulfilled in L (D), whereas $\mu_i=0$ ($\nu_i=0$) in $M\cup D\cup D_B$ ($M\cup L\cup L_B$). Also, $g_p>0$ must hold for dark (light) plaquettes in $M\cup L_B$ ($M\cup D_B$), whereas $g_p=0$ in D (L). All this can be done in such a way that the couplings vary smoothly across the surface due to the thickness of the boundary regions L_B and D_B.

As a result of the above construction, we will find in general a local degeneracy in the ground state, since there exist areas in L_B (D_B) where the only nonzero coupling is g_p in dark (light) plaquettes. This local degeneracy can be removed by letting the support of μ_i (ν_i) overlap with that of the g_p of light (dark) plaquettes. In doing so, the Hamiltonian is no longer exactly solvable, but it will fulfill the required conditions at least as long as the overlap is not too big. To see this, note that we can write the Hamiltonian as $H'=H+H_p$, where H_p contains those terms that do not commute with all the terms of H'. Then H is exactly solvable and has the required properties. Also, $[H_p,H]=0$. Indeed, each of the terms of H_p commutes with each of the terms in H. Thus, a small H_p will not destroy the properties of H discussed above. Still, if the overlap is too big, a level crossing could occur taking the ground state out of the subspace V described by Eq. (6).

C. Surface deformation

Once one is able to engineer the Hamiltonian (10), the next step is to adiabatically modify the couplings so that the geometry of the surface changes slowly with time. Here we can distinguish two kinds of such surface deformations. First, we can perform deformations in which only the geometry of the surface, not its topology, change with time. When

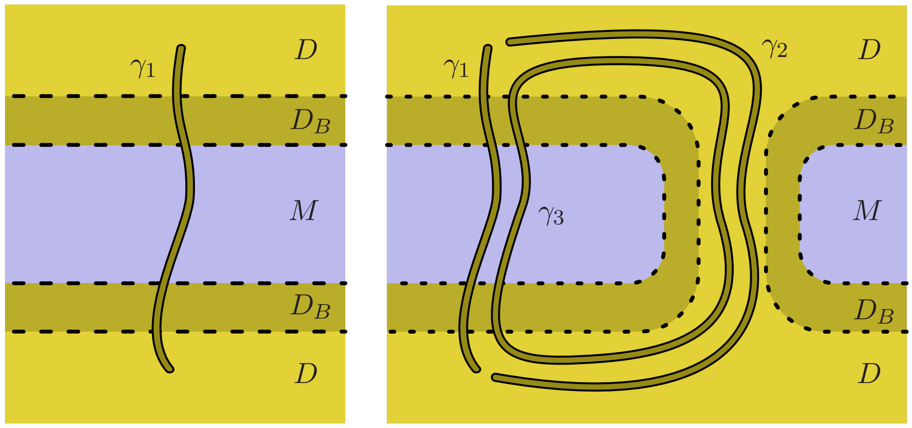

FIG. 5. (Color online) A deformation in which two separated D regions are put together, which amounts to cut the main region M. To the left, the geometry before the cut is done. We suppose that γ_1 is a nontrivial closed string. To the right, the geometry after the cut. Now γ_1 is a boundary string and so are γ_2 and γ_3. If $Z^{\gamma_2}=1$, then $Z^{\gamma_1}=Z^{\gamma_3}$, that is, the cut maps the value of Z^{γ_1} to the light plaquete charge in the region surrounded by γ_3.

the initial and the final states of the system are the same, these produce a continuous map of the surface onto itself, so that in particular strings get transformed. This gives a string operator mapping, which amounts to perform a definite operation on the encoded subsystem.[33] Second, deformations that change the topology can be considered, such as introducing or destroying holes and cutting or gluing pieces of the main surface M. These kinds of processes change the topological degeneracy of the ground state. When it grows, the new degrees of freedom will be initialized in a definite way[33] due to topological considerations. When it decreases, the lost degrees of freedom get mapped to possible excitations in the final state.

This deserves a more detailed explanation. Consider for example the surface deformation illustrated in Fig. 5, where two separate pieces of region D get connected, producing a cut in M. Consider the dark string γ that connects both D areas. We want to show that the deformation amounts to a measurement of Z^{γ}. Before the deformation γ is closed—and we assume that nontrivial—and after the deformation it is a boundary. Because of the local nature of the deformation, it cannot change the value of Z^{γ_1}, which lies outside the area where the action occurs. But if $Z^{\gamma_1}=-1$, then the final state cannot fulfill conditions (6) and thus it is not a ground state. Which excitations should we find? To answer this, let us suppose that the coupling μ_i is big enough in D, so that in the final state we know that $Z^{\gamma_2}=1$ is fulfilled for any dark string γ_2 lying inside D. Then for the dark boundary string γ_3 formed by composing γ_1 and γ_2, see Fig. 5, and for the final state $|\psi\rangle$ we have $Z^{\gamma_3}|\psi\rangle=Z^{\gamma_1}Z^{\gamma_2}|\psi\rangle=Z^{\gamma_1}|\psi\rangle$, since the value $Z^{\gamma_3}=\pm 1$ is related to light plaquette charge inside γ_3 through Eq. (8). We see that the cutting process, as announced, amounts to a measurement of Z_2^{γ}, as its value is mapped to the possible appearance of charge at both sides of the cut.

For the previous analysis, the deformation needs not really be adiabatic. It is enough if we can guarantee that there are no excitations inside D. The particularity of the adiabatic case is that we expect to find a final state with a single light plaquette excitation at each side of the cut, since this is a state in a local energy minimum. We will see an application of these measurements through surface cutting—and indeed of all the mentioned kinds of surface deformations—in the scheme to demonstrate the topological character of the phase discussed below.

It is worth mentioning that these ideas can be used to adiabatically initialize the topologically ordered phase. In this regard, a question was raised in (Ref. 37) about how to adiabatically initialize these systems so that the topological protection is present all along the way and not only after reaching the topological phase. The answer is that, instead of initializing the whole system at a time, one should progressively grow it from a small island till the desired surface is covered. In surfaces with nontrivial topology, this means that at some point two different borders of the system will fuse. At that point the degeneracy of the ground state will change as new nontrivial string operators appear. The eigenvalues of the new string operators that run along such junctions are necessarily one,[33] and thus the final particular ground state of the system is perfectly determined.

IV. SCHEME FOR DEMONSTRATING TOPOLOGICAL ORDER

When trying to demonstrate TO, the usual approaches focus on interferometric experiments with quasiparticles in which topologically different paths are compared. An immediate problem of such approaches is that the required quasiparticle superposition of states are subject to decoherence due to their localized nature and the presence of a noisy environment. Also dynamical phases have to be taken into account and properly controlled. Here we adopt a different approach that eliminates both problems by focusing on the ground-state degeneracy. The idea is to show that the outcome of certain processes depends only on topological properties, thus revealing the topological nature of the system.

The scheme is as follows. We start by making a pair of holes in our system, a dark one and a light one [see Fig. 6(a)]. Then we deform both of them as in Fig. 6(b) till they are separated into two pieces. Notice that since in figure Fig.

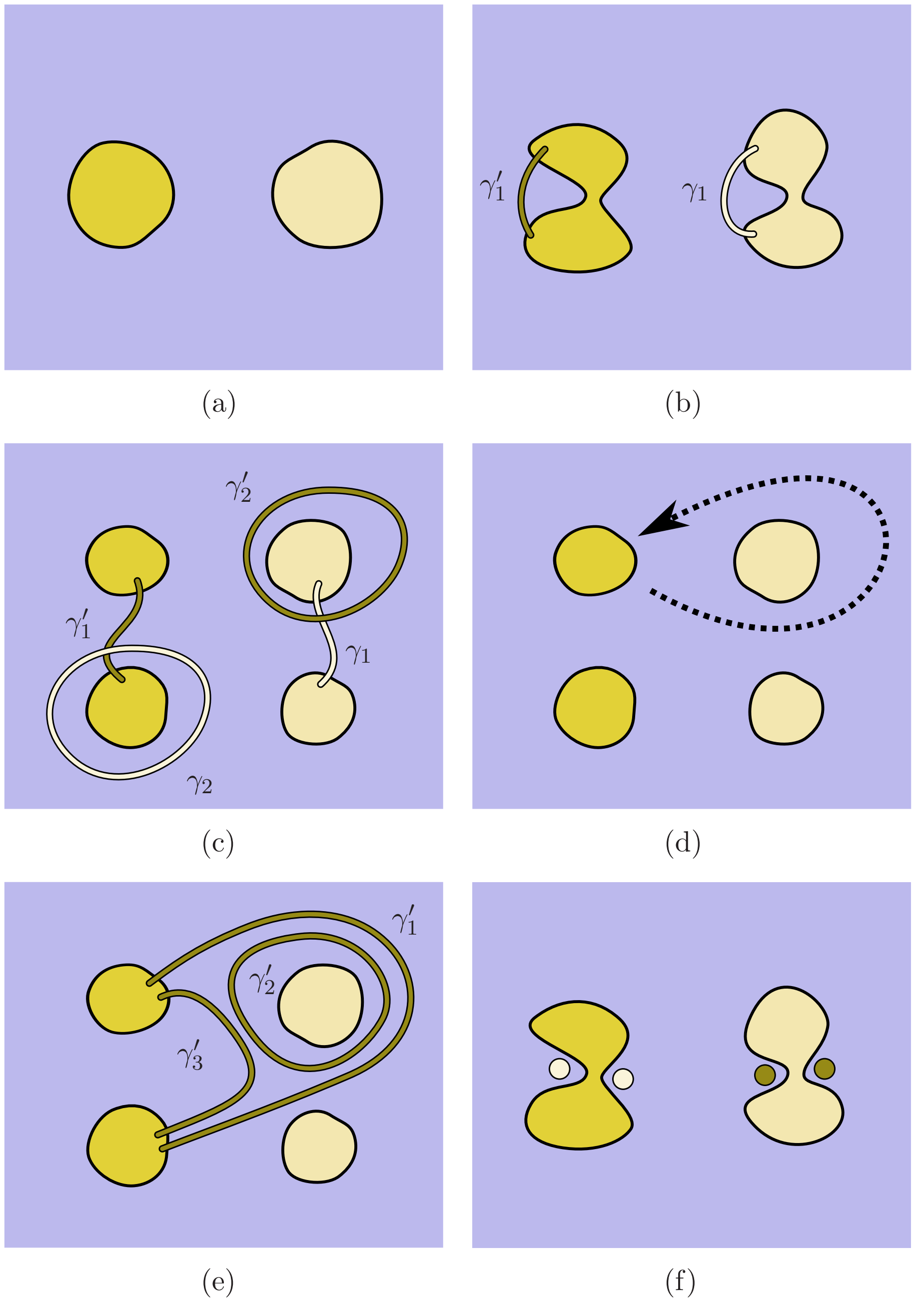

FIG. 6. (Color online) A step-by-step representation of the proposed scheme, as explained in Sec. IV.

6(b) γ_1 and γ_1' are boundaries we have $X^{\gamma_1}=Z^{\gamma_1'}=1$. After the hole breaks into two pieces they still must have the same value because it is a global property,[33] so that we reach the situation in Fig. 6(c), where X^{γ_2} and $Z^{\gamma_2'}$ have completely undefined values since $\{X^{\gamma_1},Z^{\gamma_2'}\}=\{X^{\gamma_2},Z^{\gamma_1'}\}=0$. We then proceed to move one of the dark holes along a closed path. Suppose for the moment that the path is as the one shown in Fig. 6(d), that is, that it encloses one of the light holes. The point is that, after this has been accomplished, the string operators have deformed accordingly. For example, γ_1' has changed and now its place is occupied by γ_3' [see Fig. 6(e)]. If $|\psi\rangle$ is the state corresponding to that figure, we have $Z^{\gamma_3'}|\psi\rangle=Z^{\gamma_1'}Z^{\gamma_2'}|\psi\rangle=Z^{\gamma_2'}|\psi\rangle$. A similar analysis holds for a light string connecting the light holes. When we finally refuse the holes, as in Fig. 6(f), we are measuring these string operators that connect each pair of holes, which have a completely undefined value, so that there exists a 1/2 probability that we find charges at both sides of the fusion point, as follows from the explanation in Sec. III C. The problem of how to detect this charge would depend on the particular experimental situation.

Now return to the path in Fig. 6(d) and consider any line l joining both light holes. We can imagine many other closed paths, some of them never crossing this line and others crossing it many times. The topological property in which we are interested is the number of times a path crosses l. If the number is odd, the situation is the one described above. If it is even, then it is equivalent to doing nothing and if we refuse the holes we will never find charges.[33] Also, note that several quasiparticles could be created during the fusion of the holes if it is not adiabatic, but the evenness or oddness of the number of particles created at each side is topologically protected since it gives the total topological charge.

Thus the topological nature of the system manifests in the fact that the experiment is sensitive to the topology of the chosen path. Moreover, the underlying Z_2 nature of the system is revealed also: only the evenness or oddness of the linking number is important. With this scheme we have introduced a way to probe the existence of a TO. It does not involve the ability to manipulate individual quasiparticle excitations, but instead relies solely on the peculiar ground-state properties of topologically ordered quantum systems.

V. FINAL REMARKS

Although we have restricted ourselves to the Kitaev Z_2 model, it is possible to consider generalizations to Z_D systems or even non-Abelian models. In this regard, as noted above, the definition of holes in terms of open strings is a natural starting point and a much richer family of "holes" is expected in such systems, but the basic mechanism for topological detection without resorting to quasiparticle interferometry remains the same.

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